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Forecasting the number of vehicle kilometers by applying the autoregression model, using Warsaw trams as an example

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Abstract: One of the biggest challenges of the 21st century is ecological responsibility. It also concerns the sustainable development of transport and the reduction of threats related to the negative impact of this phenomenon on the environment. A constant increase in transport congestion, atmospheric air pollution, and noise promotes the search for new solutions, especially in urban areas. One of the systematically implemented and improved ideas in this area is the development of urban transport systems. Their effectiveness and efficiency are evidenced by the level of meeting the transport needs of residents, with the optimal utilization of vehicles. The article analyses urban transport in Warsaw, focusing only on trams as the second most popular means of transport after wheeled vehicles. Two objectives of the study were adopted. The first was evaluating the current state and characteristics of the available options and indicating potential development directions, considering factors that determine it. The second goal was to select the appropriate model describing the number of vehicle kilometers accumulated by Warsaw trams in the years 2017-2019 and parametric identification of this model. The study allowed us to estimate and make a short-term forecast of transport services carried out by trams. The research has shown that the current situation regarding the performance of transport work by trams in Warsaw does not fit into the paradigm of sustainable transport development. This is due to the loss of vehicles from the records in the absence of new vehicle purchases. Additionally, the developed tool indicates a decrease in the number of vehicles-km performed in the following months and, thus, a reduction in the share of trams in transport in the Warsaw communication system. The identified problem (i.e., a downward trend in transport performance) is essential from the point of view of the quality of the system's operation and the ability to meet passengers' expectations. It also informs decision-makers about the need to implement changes leading to an increase in the share of tram transport, mainly in capacity and operating costs.

Keywords: autoregression model, sustainable transport, forecasting

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1. Introduction

Urban transport systems play a significant role in shaping socioeconomic relations. Their development should be carried out following the paradigm of sustainable transport development, as well as in a way that fully meets the transport needs of residents and their quality requirements, assuming the optimal use of transport means. Not only the level of transport quality must be constantly raised, but also the method of management that will allow it to match passengers' expectations best.

This article presents an evaluation of the current state. It indicates potential development directions of the rolling stock of tram vehicles in Warsaw. It offers an analysis of the vehicle-km realized by trams with the development of a model for predicting this phenomenon in the future. The study was carried out based on dynamic ARIMA class models, which can be used in the case of non-stationary time series, taking into account the changes in the relations that shape the transport process. In addition, the method enables detailed analyses to be carried out in a situation where there is no thorough knowledge about the studied phenomenon (in this case, insufficient documentation of the tasks performed by the operator).

2. Literature review

Public transport is one of the critical elements conditioning every agglomeration's economic and social progress. Sustainable development, preceded by forecasting the volume of transport work, promotes better functioning and increases reliability [1]. Additionally, it positively influences the reduction of transport congestion and the accompanying air pollution and noise level. Underestimation of the transport needs makes it difficult for city dwellers to move about and may result in such services being abandoned [2]. On the other hand, overestimation is associated with financial losses and an unjustified increase in investment outlays.

In the literature, the studies of the transport work volume of public transport mean most often concern the estimation of the long-term forecasts as the instruments supporting decision-making in the field of infrastructure investments management. To this end, multifactorial regression models (MLR), artificial neural networks (ANN), or principal component analysis (PCA) are used, as well as combined PCA-MLR and PCA-ANN models [3].

The second group of the research conducted is short-term forecasts, which are the subject of this article [4], [5]. Their purpose is to support the day-to-day management of the available infrastructure and suprastructure, minimizing the adverse effects of increased traffic and optimizing the use of existing rolling stock. The models of moving average and exponential smoothing [6], [7] and multifactorial regression [8], [9] are used, as well as methods based on the model with excess zeros [10], Markov models [11] and machine learning techniques [12]. In the case of significant variation in traffic intensity, nonparametric methods, such as artificial neural networks [13], support vector machine (SVM) [14], or Kalman filter methods [15], are used. Li et al. have presented a method of forecasting the volume of transport work based on a particle swarm algorithm [16]. At the same time, Sun et al. proposed a forecast model based on neural networks with the radial base function (RBF)

[17]. Gu et al., in turn, conducted research based on the improved Bayes model [18], and Wang et al. developed a wavelet network model [19]. The studies are dedicated mainly to wheeled vehicles. This article analyzes and evaluates the volume of tram transport work in Warsaw, an essential element of the public transport system (30% share in all transports). In addition, they are characterized by a higher capacity than wheeled vehicles (the maximum length of a bus is 24 m, and a tram is 76 m) and lower fuel consumption due to the infrastructure on which they travel (rails versus paved asphalt road). The place where the research is conducted is also essential, i.e., the capital city of Warsaw, for which no similar analyzes have been completed. However, the increase in the number of people and the number of vehicles registered on its territory, as well as the increasing inflow of inhabitants of the agglomeration commuting to the city center by private cars every day, necessitates taking measures to make traveling by public transport more attractive. One possibility is the development of models to help manage the distribution of transport performance in the city. Few tools in the literature will be based mainly on historical observations as having the most significant impact on shaping the studied phenomenon in the future. Therefore, in the article, concerning the analysis of the transport performance of trams in Warsaw, the dynamic autoregressive integrated moving average (ARIMA) model was proposed, enabling the development of a model describing the dependence of a variable in time and the extrapolation of the time series into the future.

3. Research method and methodology

In a situation where detailed knowledge about the analyzed process is not available, but only information describing the course of the examined variable in the past, the time series forecasting methods are applied in which the previous observations are used [5], [9]. They make it possible to develop a model describing the dependence of a variable over time and extrapolate the time series into the future.

The most important and widely used in this area are autoregressive moving average (ARMA) class models. ARMA models can represent several different types of time series, including pure autoregression (AR) or pure moving average (MA).

The autoregressive processes AR (*p*) (autoregressive model) of the order of $p \in N$ describe time series in which there is a significant autocorrelation between the examined variable and its time-delayed values [20], [21]. The identification of such a model comes down to determining the level of differentiation, i.e., the order of autoregression (of the *p* parameter). The equation of the autoregression model of the p order [21], [22] is in the form:

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \dots + \alpha_p x_{t-p} + \varepsilon_t \tag{1}$$

where:

$\{\varepsilon_t\}_{t \in N}$ - a series of independent random variables with the distribution $N(0, \sigma^2)$,

α - the parameters of the autoregressive part of the model.

In the autoregression models, the value of observations at the t moment is the sum of the random component and previous observations of the examined variable [23]. If the value of a random element is a function of its time-delayed values, then we are dealing with the moving average MA (*q*) process [24], [25]. In this case, the identification of such a model comes down to determining the order of the moving average process, i.e., the $q \in N$

parameter. It informs about the number of delays included in the equation. The moving average model is:

$$x_t = \varepsilon_t - \beta_1 \varepsilon_{t-1} - \beta_2 \varepsilon_{t-2} - \dots - \beta_q \varepsilon_{t-q} \tag{2}$$

where:
$\{\varepsilon_t\}_{t\in N}$ - a series of independent random variables with the distribution $N(0, \sigma^2)$,
β - the parameters of the moving average part of the model.

The ARMA model combines the autoregressive model and the moving average. The basis for its identification is the assumption that both the past values and the values of the time-delayed random component affect the value of the examined variable at time t. The equation of the ARMA model is in the form [21], [26]:

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \dots + \alpha_p x_{t-p} + \varepsilon_t \beta_0 + \varepsilon_t - \beta_1 \varepsilon_{t-1} \\ -\beta_2 \varepsilon_{t-2} - \dots - \beta_q \varepsilon_{t-q} \tag{3}$$

The ARMA class models are used in the case of stationary series. We say that the time series is strictly stationary if the cumulative and conditional process probability distributions do not change over time. Weak stationarity (in a broad sense) means constancy during the first and second-order moments. The covariance for any two observations is also constant, depending only on the distance between them, not the observation moments.

It is possible to bring non-stationary series to the stationary form using a differential operator [21], [27], [28]. Then we are talking about a time series integrated into d degree where d is the differentiation order [21], [29]. In the stationarity testing, the most common are unit root tests: the extended ADF Dickey-Fuller test [30] (Augmented Dickey-Fuller test), Kwiatkowski, Phillips, Schmidt, and Shin test (KPSS test) [31], [32]. This article uses the ADF test. The null hypothesis assumes that the studied time series is non-stationary. The alternative hypothesis assumes that the time series is stationary. The test statistics have the Dickey-Fuller distribution and have a form:

$$DF = \frac{\hat{\theta}}{S(\theta)}, \tag{4}$$

where:
$\hat{\theta}$ - the estimator of the θ parameter, while $S(\theta)$ is the standard deviation of this parameter.

Suppose the time series is non-stationary but can be brought to a stationary form by differentiation. In that case, this series is identified through ARIMA (p, d, q) models.

This article assesses the monthly transport performance by trams in Warsaw, expressed in vehicle-km, from January 2017 to September 2019. For the following study, a research hypothesis was formulated, which says that the development of a model for predicting the amount of transport performance to be performed by trams will positively affect the proper management of the rolling stock, which will enable better meeting the expectations of passengers, reduction of transport congestion and minimizing the adverse effects of transport on the environment. Two main research goals were also adopted, i.e., the evaluation of the current state, characteristics of the available options, and indicating potential development directions, taking into account factors that determine it and selecting the appropriate model describing the number of vehicle kilometers accumulated by Warsaw trams in the years 2017-2019, and parametric identification of this model.

4. Characteristics of the test object

One of the elements of sustainable public transport development is the use of ecological means of transportation. It is crucial to systematically replace buses with high-combustion diesel engines still in use in many cities and with significant environmental impact.

In addition to introducing a modern fleet powered by compressed natural gas, which meets the restrictive EU Euro VI emission standard, investing in electric vehicles is also a priority. Among them, we can distinguish trolleybuses and electric buses, as well as trams being the subject of this analysis. The increase in the size of the fleet of electric vehicles perfectly fits into the trend of limiting the use of transport means powered by fossil fuels in favor of modern solutions with lower emissions of both pollution and noise, which was imposed by the policy of the European Commission and is consistent with the Strategy for Sustainable Transport Development up to 2030 [33].

Even though the construction of the tram line causes high initial costs resulting from the structure of the track, power supply network installation, and accompanying infrastructure, Warsaw is one of the cities that promotes investments in the development of rail transport [34]. Apart from the ecological element, this is also due to the high transport capacity of trams and significantly lower operating costs compared to buses.

Managing the tram network in Warsaw is the task of Tramwaje Warszawskie sp. z o.o. (TW), separated in 1994 from the Municipal Transport Company (MZK). The tram route network is well developed; it consists of 433 km of tracks, about 90% separated from other traffic. Twenty-six tram lines are running on them, served by 726 cars [34].

The change in the potential of TW rolling stock in the years 2012 - 2020 is presented in Figure 1. It presents the number of tramcars owned in a given year, including deletions from the records and purchases, which are additionally shown in the chart.

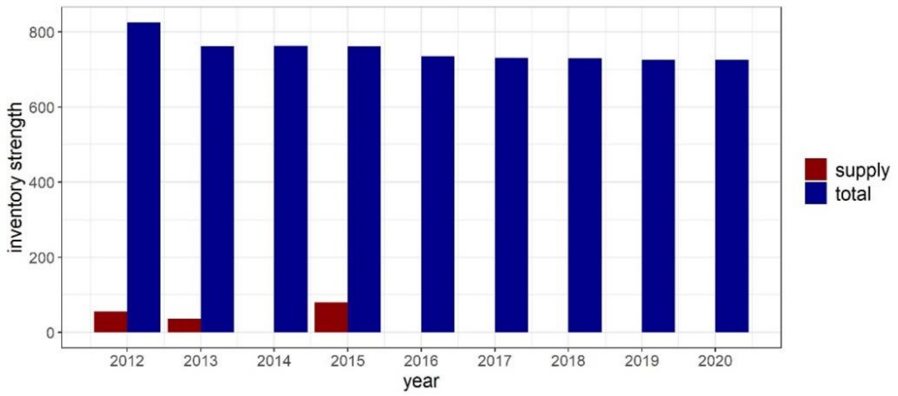

Fig. 1. Record status of TW rolling stock in 2012-2020

As shown in Figure 1, the rolling stock of Warsaw trams is not characterized by an increase consistent with expectations resulting from the Strategy for Sustainable Transport Development up to 2030 [33], which assumes in 2015-2030 an increase in the tram transport work (in millions of passenger-kilometers) in the urban transport up to 114% and an increase in the volume of public transport trips (in millions of persons) up to 102%. However, the number of transport means owned does not unequivocally determine the lack of progress. Effective management of the rolling stock owned and optimization of transport adapted to the needs of inhabitants may also ensure this. It increases interest in this form of mobility and can influence the decision to switch individual journeys to public transport.

5. Results

The research sample consisted of 33 observations, aggregated for each month. Since the number of days in individual months of the calendar year is not constant, the average value was used for the analysis. The model describing the analyzed phenomenon was identified, and estimates were made of the future time series values. The AR model was used for this purpose. The graph of the studied time series is presented in Figure 2, illustrating the variability of the studied phenomenon.

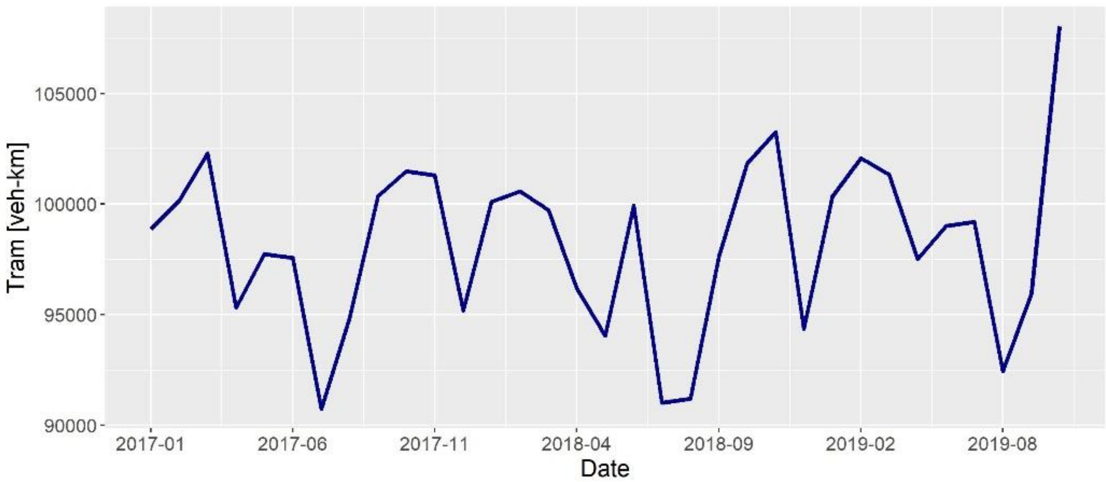

Fig. 2. Graph of the average monthly values of veh-km achieved by Warsaw trams

Considering the above, an attempt was made to develop a model describing the number of vehicle-km provided by trams along with its parametric identification, as well as to use this tool to develop short-term forecasts supporting the management of the division of the transport performance. The average number of transports carried out varies from month to month. The highest value was achieved in September 2019. The remaining selected descriptive statistics are presented in Table 1.

Table 1. Basic descriptive statistics for monthly average vehicle values in veh-km

Min.	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max.**
90739	95314	99015	98231	100588	108037

The variable distribution is normal (Figure 3). The value of the Lilliefors test statistic is *D* = 0.112 and *p-value* = 0.363.

In the next step, the stationarity analysis was performed using the extended Dickey-Fuller test. The value of test statistics is *DF*=-2.685, while *p-value* = 0.308. Therefore, at the $\alpha =$ 0,05 level of significance, there are no grounds for rejecting the hypothesis regarding the non-stationarity of the series, which is illustrated in Figure 4. The statistically significant values appear on the autocorrelation function graph.

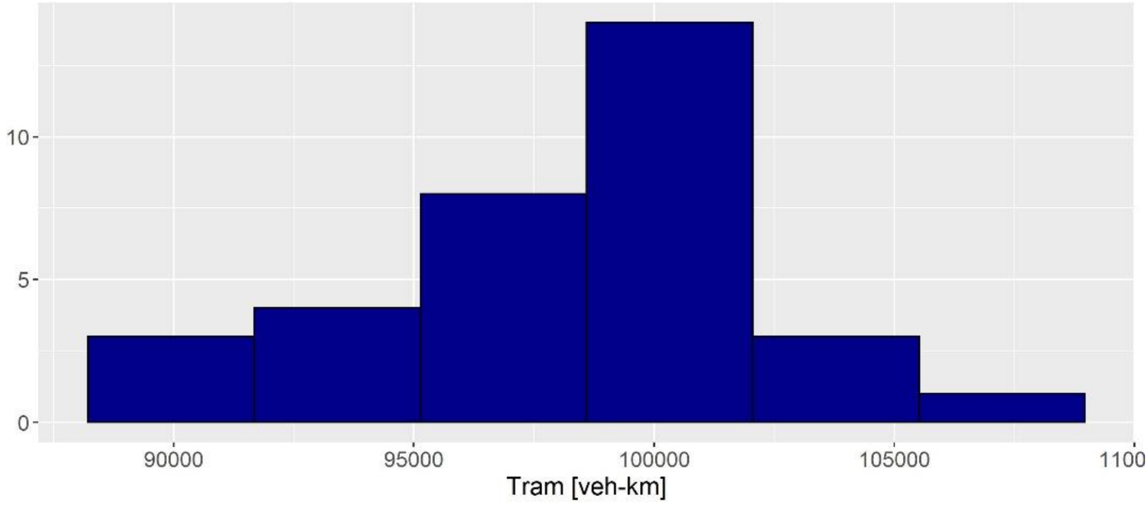

Fig. 3. Histogram of the examined variable

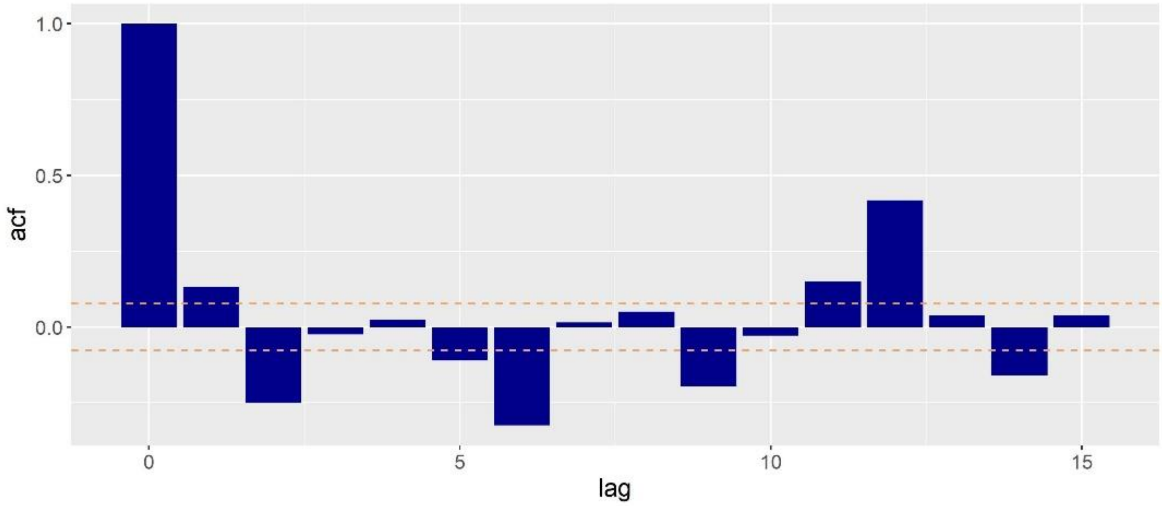

Fig. 4. Graph of the autocorrelation function (ACF) of the examined variable

Only after one differentiation and reusing of the ADF test was it found that at the level of significance $\alpha = 0{,}05$, the hypothesis of series non-stationarity can be rejected ($DF = -4.329$, *p-value* = 0.01).

The AR (1,1) model was used to describe the implementation of the tram transport in the form:

$$\Delta x_t = -0.1896\, \Delta x_{t-1} + \varepsilon_t, \tag{5}$$

The information criteria for this model are AIC=636.92 and BIC= 639.85 (where AIC - Akaike Information Criterion and BIC - Bayesian Information Criterion). Afterward, a short-term forecast of the studied phenomenon was developed. The graph of the examined series and projections are presented in Figure 5.

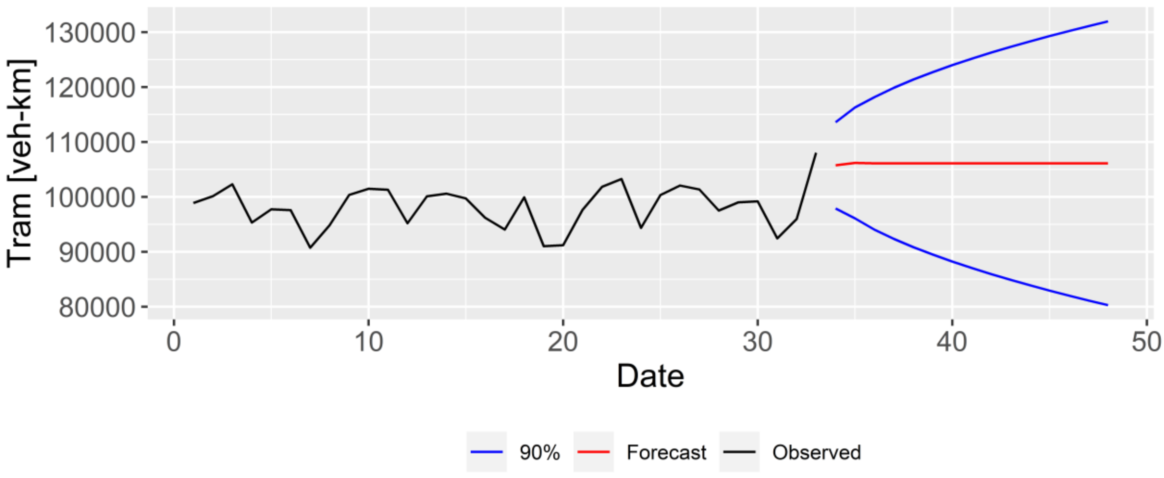

Fig. 5. Graph of the examined series and forecasts

Estimating model parameters and their verification allows us to use them for predictive purposes. There is no need to obtain additional information about the process because the future value of the forecast variable is obtained based on its value from previous periods, which is a significant advantage of the presented approach. It will allow estimating the number of vehicle-km covered by Warsaw trams in the coming months. From the estimated function, it follows that the value of the forecast observation is smaller than the previous one, which is not an expectation consistent with the directions postulated as part of urban transport development. However, this result may be affected by the fact that the survey was conducted on aggregated monthly data, as Tramwaje Warszawskie Sp.z o.o provides such databases. Therefore, as part of future research, it is worth analyzing daily and even hourly data. This would allow us not only to determine more precisely how the use of trams changes over time but also to observe the likely seasonality of the process.

In addition, the disadvantage of making forecasts based on historical data is the lack of resistance to external disturbances caused by the occurrence of new phenomena and the possibility of using distorted results that some previously unexplored factors have distorted. Such a situation may occur, for example, when examining only a single process under specific conditions without reference to similar processes occurring elsewhere, which can affect the shape of the forecast. In the case of this article, the estimates may be affected by internal factors, such as signed contracts for the delivery of new rolling stock, which may be carried out as planned or have delays, or various types of decisions by rolling stock managers. It is also essential to keep in mind the possibility of new external factors that did not affect the historical data. Still, it will now be of great importance - such as the emergence of a pandemic or increased demand for communications due to the immigration of war refugees.

6. Conclusions

One of the main objectives of modeling is to determine the regularity of the changes level of the phenomenon under study over time. Dynamic models, considering the delayed value of the dependent variable, are used in this scope. In this article, the autoregressive model has been used.

The literature analysis has shown that the methods of forecasting the volume of transport performance of public transport means are a frequent research topic. The authors use a wide range of tools for this purpose. The research conducted in the article showed the possibility of using ARIMA methods to forecast the number of wagon kilometers performed by public transportation. Given the high variability of public transportation traffic, nonparametric methods are more suitable for making short-term forecasts. The use of the ARIMA model in this study made it possible to effectively describe the time dependence of a variable and extrapolate the time series into the future, creating a short-term forecast of Warsaw Trams' traffic work. The study results show that this method effectively adapts infrastructure and rolling stock management strategies to current and future traffic conditions by optimizing available resources. The proposed solution leaves room for further research. The authors could compare the methods of making forecasts presented in the literature review with the one described in this paper to assess its effectiveness and show the differences in the results obtained.

An intensively changing urban environment requires constant adaptation to its processes - including relocation. The growing needs in this respect are a challenge for public transport, whose main goal should be to manage the existing rolling stock and the quality of transport in such a way as to be competitive with individual journeys. Increasing the number of public transport users and the residents declining to travel in their vehicles will not only reduce congestion but also reduce the negative impact on the environment. Therefore, investing in ecological forms of public transport is desirable, which, without a doubt, include trams. Contrary to the expectations, the analysis performed does not fit into the paradigm of sustainable transport development concerning the capital city. The investments in this area are not substantial. The purchases of new cars are smaller than their losses from the records. Additionally, monthly, the model developed indicates a decrease in the next value of the series concerning past observation. However, this may be due to the seasonality of the process or other factors whose impact is not visible due to the data aggregation. Therefore, to have in-depth diagnostics as part of future tests is to analyze the transport work carried out daily or hourly. It is also worth considering evaluation taking into account the administrative division of Warsaw, which will allow identifying the areas with different intensities of inhabitants' movement. The above situation may also be improved by contracts signed in 2019 for the purchase of additional transport means in the form of 123 trams, as well as the planned infrastructure development by constructing about 27 km of routes, creating new connections with two numerously populated districts of the city, i.e., to Goclaw and Wilanow. The completion of the above activities is planned for the end of 2023.

The identified problem (i.e., a downward trend in transport performance) is essential from the point of view of the quality of the system's operation and the ability to meet passengers' expectations. It also informs decision-makers about the need to implement changes leading to an increase in the share of tram transport, mainly in capacity and operating costs.

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