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ANALYSIS OF MULTICRITERIA TRANSPORTATION PROBLEM CONNECTED TO MINIMIZATION OF MEANS OF TRANSPORT NUMBER APPLIED IN A SELECTED EXAMPLE

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Abstract: This article presents the method connected to formulation and solving multicriteria transportation problems in the context of necessary transportation resources. In the developed method transport resources are considered in the aspect of their number minimization. Moreover, authors of the article presents several well-known methods for solving multicriteria transportation problems, and then develop solutions of multicriteria linear transportation problems for minimization of the number of transport resources (means of transport). Furthermore, authors describes an example of a solution as a result of using an original computer application. What is more, the development of correctness verification of the presented algorithm has been conducted. The article was concluded with a summary along with an indication of further work in the subject matter.

Keywords: transport, transportation problems, operational researches, multicriteria decision making, multicriteria optimization

INTRODUCTION

In the classic approach, the multicriteria transportation problem aims to define the transportation schedule of cargo supplies between certain quantity of multiple sources to a certain quantity of destinations, taking into account multicriteria analysis. Determination of the optimal transportation plan is very important from the point of view of any concerned organization. Proper management of this supply process can significantly lead to reduce the cost, as well as transportation time, which can bring significant advantages. However, the multicriteria transportation problems are complex and demanding highly computational capacity and often an individual approach in order to determine a satisfactory solution. A selection of an adequate method in order to optimize the transportation plan is a complex issue. There are plenty of methods which help to solve multicriteria transportation problems, which significantly vary from each other both in the way of algorithms method and the approach of their formulation.

The majority of multicriteria transportation problems are analyzed in the context of the cargo streamed to particular routes. The key target in such type of problems is to determine transport quantities, for which the criteria reach their Pareto optimal values, whilst constrains are met (Pareto optimal solution is an allocation of available goods or resources, so that they could be used in the maximum way; Pareto optimal solutions are obtained when an attempt to improve the value one of one criterion causes lower quality or degradation of any other criteria – own definition based on Mornati (2013). In those type of the problems criteria usually are connected to cost, time and/or distance of transportation. However, according to literature review this approach does not include the number and the types of means of transport. Both of them (the number and types) may play a significant role in the transportation processes.

Determination of the transportation plan from the point of view of cargo volume implies the necessity of using additional tools (methods) to assign means of transport in order to carry out the transport. Because of that fact, it seems rational to formulate multicriteria transportation problems in the context of the means of transport number. This aims to assign a minimum number of means of transport of different types to particular routes in such a way that both the number of transport modes and the cost, time and distance are minimized.

Such an approach covers the topic of multicriteria transportation problems in a holistic way. In other words, it covers a full range of possible criteria and it returns the optimal transportation plan from the perspective of various means of transport types.

1. ANALYSIS OF THE LITERATURE OF MULTICRITERIA DECISION-MAKING PROBLEMS IN THE CONTEXT OF TRANSPORTATION PROBLEM

The variety of multicriteria decision-making problems causes that they are often presented by different authors. Research connected to them are realised in such areas as production as in Żelazny (2012), Matuszek and Kurczyk (2011), industry as in Pempera and Żelazny (2015), logistics as in Ho et al. (2010), Wang et al. (2011), Ambroziak and Lewczuk (2009), Pyza (2011), transport as in Alaia et al. (2015), Jacyna and Turkowski (2014), Jacyna and Wasiak (2015), environmental protection as in de Roo et al. (2012), as well as medicine as in Küfer et al. (2005), Muzalewska and Moczulski (2016).

There are also many works aimed to classify methods of solving multicriteria decision-making problems. One of the classifications proposed by the authors of Hirche (1980) structured them depending on the moment of involvement of the decision maker in the process of the problem solving. The authors of Hirche (1980), Malesa (2012) distinguished four categories of methods:

– Methods in which the decision maker is not involved in the solving process and his/her preferences are not taken into consideration,

- „*A priori*" methods, in the case of which the decision maker is involved at the beginning of the decision-making process, expressing the preferences and initial assumptions,
- Interactive methods, in which a decision maker is involved during the entire decision-making process, is informed about the individual stages of problems solving and depending on the partial results – a decision maker may change assumptions at various stages of solving the problem,
- „A posteriori" methods, in which usually a set of Pareto optimal solutions is generated without involvement of decision maker, who only at the end of the process choose one of them as the final solution.

Furthermore, the multicriteria problems are often classified depending on the criteria. The author of Ehrgott (2005) divides them into:

- Scalarization techniques, in which criteria are combined in one and then solved with the use of one-criteria optimization methods,
- Pareto techniques, in which the solution usually consists of a set of Pareto optimal solutions.

There are many methods (and their classification) of solving multicriteria decision-making problems (their review is included in the article Trzaskalik (2014). This mainly results from the complexity and diversity of issues. In turn, this implies the necessity of individual approach to individual issues and each time searching for new, unconventional methods of solving them.

In the literature, there are also a number of publications concerning the multicriteria transportation problems and methods of solving them. However, in all these publications, multicriteria transportation problems are considered in the context of the volume of cargo.

The proposed methods of solving multicriteria transportation problems can be divided into three categories. Methods using fuzzy programming – as in Bit et al. (1992), Doke and Jadhav (2015), Gupta and Kumar (2012), Muruganandam and Srinivasan (2016), genetic algorithms – as in Alaia et al. (2015), Bharanti and Vijayalakshmi (2016), Zaki et al. (2012), Gen et al. (1999) and methods finding the entire set of efficient (Pareto optimal) solutions using multicriteria linear optimization methods – as in Isermann (1979), Diaz (1979), Nair and Aneja (1979), Shi (1995) or simulation techniques and methods, e.g. Kostrzewski (2018).

2. FORMULATION AND METHODS OF SOLVING MULTICRITERIA TRANSPORTATION PROBLEMS

A complex nature of multicriteria issues makes it difficult to formulate a universal mathematical model for "all" multicriteria transportation problems. Each of them presents a different intention dictated by assumptions and data, which (depending on the considered issue) retakes different interpretation.

Because of this reason, the formulation of the multicriteria transportation problem is presented in this paper for three criteria looking for extremes in the aspect of: costs, time and distance of cargo transportation. This form is presented below and indicated according to the equations (1) – (16).

The data in the transportation problem are as follows:

– Set of indexes of homogeneous cargo sources

$$\mathbf{D} = \{ 1, 2, 3, \dots, i, \dots, D \} \tag{1}$$

where $i \in \mathbf{D}$ is the number of i-th source; D – total number of sources,

– Set of indexes of destinations for homogeneous cargo

$$\mathbf{O} = \{ 1, 2, 3, \dots, j, \dots, O \} \tag{2}$$

where $j \in \mathbf{O}$ is the number of j-th destination; O – total number of destinations,

– Set of criteria indexes

$$\mathbf{P} = \{ 1, 2, 3, \dots, p, \dots, P \} \tag{3}$$

where $p \in \mathbf{P}$ is the number of p-th criterion; P – total number of criteria,

– Criteria matrixes:
  – Cost matrix of one load unit of cargo transportation from i-th source to j-th destination

$$\mathbf{\Gamma} = [\gamma(i,j)] \tag{4}$$

  – Time matrix of one load unit of cargo transportation from i-th source to j-th destination

$$\mathbf{\Delta} = [\delta(i,j)] \tag{5}$$

  – Distance matrix of one load unit of cargo transportation from i-th source to j-th destination

$$\mathbf{E} = [\varepsilon(i,j)] \tag{6}$$

– For each i-th source, the delivery vector is given in the following form

$$A^{\mathrm{T}} = [\alpha(i)]^{\mathrm{T}} = [\alpha(1), \alpha(2), \alpha(3), \dots, \alpha(i), \dots, \alpha(D)];\ i \in \mathbf{D} \tag{7}$$

– For each j-th destination, the demand vector is given in the following form

$$B^{\mathrm{T}} = [\beta(j)]^{\mathrm{T}} = [\beta(1), \beta(2), \beta(3), \dots, \beta(j), \dots, \beta(O)];\ j \in \mathbf{O} \tag{8}$$

The global criterion function

$$F = \langle F_1(\mathbf{\Gamma}, \mathbf{X}^*), F_2(\mathbf{\Delta}, \mathbf{X}^*), F_3(\mathbf{E}, \mathbf{X}^*) \rangle \rightarrow min(max) \tag{9}$$

However, in general, the global criterion function is as follows

$$F = \langle F_p(\bullet, \mathbf{X}^*) \rangle \rightarrow min(max);\ p \in \mathbf{P} \tag{10}$$

Individual criteria were indicated according to the equations (11) – (13)
where: $x(i,j) \in \mathbf{X}^*$ - the set of variables of multicriteria transportation problem

$$F_1(\mathbf{\Gamma}, \mathbf{X}^*) = \sum_{i=1}^{D} \sum_{j=1}^{O} \gamma(i,j) x(i,j) \rightarrow min(max) \tag{11}$$

$$F_2(\mathbf{\Delta}, \mathbf{X}^*) = \sum_{i=1}^{D} \sum_{j=1}^{O} \delta(i,j) x(i,j) \rightarrow min(max) \tag{12}$$

$$F_3(\mathbf{E}, \mathbf{X}^*) = \sum_{i=1}^{D} \sum_{j=1}^{O} \varepsilon(i,j) x(i,j) \rightarrow min(max) \tag{13}$$

Constraints are presented as it is given in equations (14) – (16)

$$\forall_{i \in \mathbf{D}} \quad \sum_{j=1}^{O} x(i,j) \le \alpha(i) \tag{14}$$

$$\forall_{j \in \mathbf{O}} \quad \sum_{i=1}^{D} x(i,j) \le \beta(j) \tag{15}$$

$$x(i,j) \ge 0 \tag{16}$$

The author of Andersson (2000) divides the methods of solving multicriteria decision-making problems into two categories: requiring and not requiring determination of derivatives of criteria. The following part of the section briefly describes the subsequent methods indicated in Figure 1.

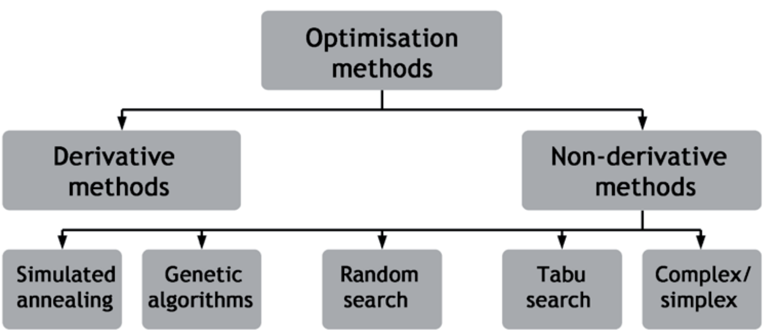

Fig. 1. Classification of multicriteria problems by Andersson (2000)
Source: Based on Andersson (2000).

Simulated Annealing is an algorithm that finds the approximate optima of the problem. Algorithm starts with generating an accidental solution, estimating its value (cost), and then generating another adjacent solution and comparing both solutions. If the new solution is better, the algorithm considers this solution and starts the procedure from scratch. Otherwise, the algorithm looks for another adjacent solution to compare it. These steps are repeated until a satisfactory solution is found or when the maximum number of repeated iterations is reached. This methods in transportation problems was applied in the paper Küçükoğlu and Öztürk (2014).

Genetic Algorithms belong to evolutionary algorithms. They are based on three main assumptions: selection, crossing and mutation of individuals (further solutions). They store a population of individual (a set of feasible solutions) in the computer memory that can cross each other (combine) and change their properties (mutate) – thus creating new individuals. Due to the fact that a given population usually has a certain maximum size, some of the weakest individuals are removed from it (selection). The (sub-)optimal solution may be the selection of the best individuals meeting the given criteria that form the next population. This methods in transportation problems was applied in the papers such as Sun et al. (2018), Saleem et al. (2016), Olteanu et al. (2018).

Random Search is a group of methods that consist of making small changes to a randomly generated initial solution, leading to new solutions. The best obtained solution becomes a new initial solution. This procedure is repeated until further changes do not lead to an improvement in the quality of the received solutions (or the searching time has elapsed) of the examined area. In the study of transportation problem, this method was applied in the paper Gdowska and Książek (2017).

Tabu Search belongs to methods similar to random search, expect that these methods enable to navigate to solutions that degrade the value of criteria. However, a sequence of previously executed movements is memorized, which are prohibited in the next steps. Thanks to this, it is possible to avoid looping and repeating cycles. The list of searched solutions is remembered and has a finite capacity. In the study of transportation issues, this method was applied in the paper Kergosien (2010).

Comprehensive methods are a limited *simplex* algorithm, consisting of several sets of possible solutions that are processed. Each set represents one point in the solution space. The idea of the algorithm is to replace the point characterized by a worse value with the point with a better value. The starting point is generated randomly, and the optimum is reached when all points in the comprehensive space are convergent. In the study of transportation issues, a method of this type was applied in the work Nourie and Güder (1994).

Hybrid Methods are methods created by combining two (or more) different methods. Usually, evolutionary algorithms are combined with sequential algorithms. The idea is to extract the best properties from each of the methods in order to determine efficient solutions for the studied problem. *Hybrid Methods* can be divided, among others, into: *Asynchronous Hybrid Methods*, *Hierarchical Hybrids*, *Additional Operators* etc. In the study of transportation issues, one of hybrid methods of this type was applied in the paper Corso and Wallace (2015).

Methods of solving multicriteria transportation problems, presented in the literature, are mainly based on three approaches (closely related to the methods described above):

- First of all, methods in which fuzzy programming are used. Fuzzy programming is used to solve multicriteria transportation problems, for which the coefficients of the criteria or constraints values are presented as fuzzy numbers. Methods that use fuzzy programing are divided into:
  - Fuzzy programming methods with a linear membership function. They consist of determining the upper and lower value (criterion) acceptable to the decision maker, determining the membership function (weights of the criterion) and solving them, most often with the help of interactive methods for solving multicriteria linear programming problems.
  - Fuzzy programming methods with the exponential (non-linear) membership function. These methods are used for problems, in which not all parameters can be described as linear form. Like in methods with the linear membership function, the upper and lower limit of the value of criteria (acceptable to the decision maker) is determined. The difference is connected to determining the exponential membership function.
- Methods using genetic algorithms to solve multicriteria transportation problems operate on the same basis as in the case of multicriteria problems of linear programming. They use natural laws of nature based on Charles Darwin's theory of evolution, and their application is based on three main pillars: selection, crossing and mutation of subsequent solutions.
- Methods using multicriteria linear optimization methods, finding the entire set of non-dominated solutions, are usually based on multicriteria linear programming methods. They often use elements of the simplex method or the theory of duality of linear programming problems.

3. METHOD OF SOLVING MULTICRITERIA TRANSPORTATION PROBLEMS IN THE CONTEXT OF MINIMIZING THE NUMBER OF MODES OF TRANSPORT

The multicriteria transportation problem, in the context of minimizing the number of means of transport (various types), differs from the classic multicriteria transportation problem in the way of its formulation. In the multicriteria transportation problem, in the context of minimizing the number of means of transport, the aim is to assign a minimum number of means of transport (various types) to the individual transportation routes necessary to carry out the transport. The criteria include both minimization of the number of means of transport, minimization of costs, time and/or distance of cargo transportation. The formulation of the multicriteria transportation problem in the context of minimizing the number of means of transport is presented for three criteria – equation (17). The adopted criteria concern minimization of the number of means of transport – equation (18), minimization of transportation costs – equation (19) and minimization of time of cargo transportation – equation (20). Designation used for dependencies (17) – (23) are as follows: $\gamma((i,j),s) \in \mathbf{\Gamma}$; $\delta(i,j) \in \mathbf{\Delta}$; $\tau\big((i,j),s\big) \in \mathbf{T}$; $x((i,j),s) \in \mathrm{C}^+$; $i \in \mathbf{D}$; $j \in \mathbf{O}$; $s \in \mathbf{S}$; $\alpha(i) \in \mathbf{A}$; $\beta(j) \in \mathbf{B}$; $\theta(s) \in \mathbf{\Theta}$; where $\gamma((i,j),s)$ – unit cost (per one kilometer) of cargo transport from i-th source to j-th destination with s-th type of the mode of transport, $\tau((i,j),s)$ – duration of cargo transport from i-th source to j-th destination with s-th type of a mean of transport; $\delta(i,j)$ – distance of cargo between i-th source and j-th destination; $x((i,j),s) \in \mathrm{C}^+$ – variable which is interpreted as the number of s-th type of means of transport directed on the route from i-th source to j-th destination; $\alpha(i)$ – has an interpretation of the supply volume of i-th source; $\beta(j)$ the demand volume of j-th destination on the specified type of cargo; $\theta(s)$ – capacity of s-th type of means of transport.

The global criteria is presented in the form of formulae

$$\langle F_1(\mathbf{\Gamma},\mathbf{\Delta},\mathbf{X}^*), F_2(\mathbf{X}^*), F_3(\mathbf{T},\mathbf{X}^*)\rangle \rightarrow min \tag{17}$$

The notations of the individual criteria is presented as formula (18) – (20).

$$F_1(\mathbf{\Gamma},\mathbf{\Delta},\mathbf{X}^*) = \sum_{i=1}^{D}\sum_{j=1}^{O}\sum_{s=1}^{S}\gamma((i,j),s)\,\delta(i,j)\,x((i,j),s) \rightarrow min \tag{18}$$

$$F_2(\mathbf{X}^*) = \sum_{i=1}^{D}\sum_{j=1}^{O}\sum_{s=1}^{S}x((i,j),s) \rightarrow min \tag{19}$$

$$F_3(\mathbf{T},\mathbf{X}^*) = \sum_{i=1}^{D}\sum_{j=1}^{O}\sum_{s=1}^{S}\tau((i,j),s)\,x((i,j),s) \rightarrow min \tag{20}$$

Sources and destinations constraints are presented in the form of formula (21), (22), with an additional notation (23).

$$\forall_{i\in\mathbf{D}} \quad \sum_{j=1}^{O}\sum_{s=1}^{S}\theta(s)\,x((i,j),s) \le \alpha(i) \tag{21}$$

$$\forall_{j\in\mathbf{O}} \quad \sum_{i=1}^{D}\sum_{s=1}^{S}\theta(s)\,x((i,j),s) \le \beta(j) \tag{22}$$

$$\forall_{i\in\mathbf{D}}\ \forall_{j\in\mathbf{O}}\ \forall_{s\in\mathbf{S}}\ \ x((i,j),s) \in \mathrm{C}^+ \tag{23}$$

The method of solving multicriteria transportation problems in the context of minimizing the number of means of transport (of various types) is connected with a directed search of a set of feasible solutions in the space of criteria, Malesa (2012). This method uses the properties of dual linear programming tasks in order to find efficient solutions. It is based on the statement that efficient solutions are interrelated and it is possible to determine them by applying appropriate mathematical transformations. The final solution is a set of efficient solutions, which (in some cases) may be a set of alternative solutions.

The method of solving multicriteria transportation problems in the context of minimizing the number of means of transport consists of three main phases.

The first phase is used to check whether the set of feasible solutions is a non-empty set ($\mathbf{N}^D \neq \emptyset$) and whether the issue under consideration has a solution.

The second phase is used to check whether the set of efficient solutions is a non-empty set ($\mathbf{N}^E \neq \emptyset$) and whether the issue under consideration has an efficient solution. The efficient solution of a multicriteria problem is connected with finding such a non-dominated point, in which the improvement of the quality of one criterion causes degradation of at least one of the other criteria. Usually, there are more than one efficient solution and they form an efficient set. In this phase, the first efficient solution is determined $N_1^E \in \mathbf{N}^E$. It determines the further calculation process.

In the third phase, the process of determining the remaining efficient solutions is realised. For this purpose, appropriate mathematical transformations and the properties of efficient solution bases are assigned.

Fig. 2 presents a block diagram of the algorithm for solving multicriteria linear problems in the context of minimizing the number of means of transport. In turn, an example of using the method is presented in the next section of the article. To find a solution, the software W.O.Z.T. was used. Software verification is presented in the next section of the article.

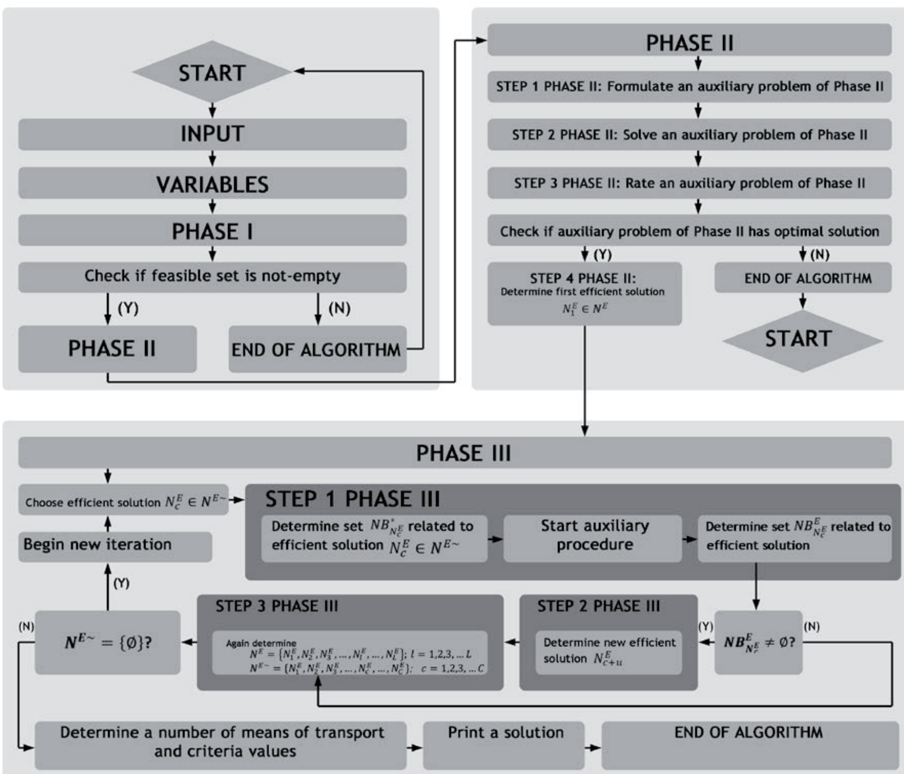

Fig. 2. Block diagram of the algorithm method of solving multicriteria linear transportation problems in the context of minimizing the number of means of transport (of various types)
Source: own study.

4. IMPLEMENTATION OF A MULTICRITERIA TRANSPORTATION PROBLEM IN THE CONTEXT OF THE MINIMUM NUMBER OF MEANS OF TRANSPORT

For four sources, three destinations, three types of means of transport and three criteria (including: minimization of the number of means of transport, costs of transport and duration of cargo), determine the optimal number of means of transport, necessary to carry out the transport of goods. Matrixes of unit costs of cargo transportation $\gamma((i,j),s)$ expressed in monetary unit [PLN], duration of cargo transport $\tau((i,j),s)$ expressed in unit [h], transport distance of the cargo $\delta(i,j)$ expressed in unit [km] and capacity vectors for particular types of means of transport $\theta(s)$ (expressed in unit [pcs.]), supply of $\alpha(i)$ sources (expressed in unit [pcs.]) and demand of destinations $\beta(j)$ (expressed in unit [pcs.]), are presented as formula (24) – (29).

$$\mathbf{\Gamma} = [\gamma((i,j),s)] = \begin{bmatrix} \gamma((1,1),1) & \gamma((1,1),2) & \gamma((1,1),3) \\ \gamma((1,2),1) & \gamma((1,2),2) & \gamma((1,2),3) \\ \gamma((1,3),1) & \gamma((1,3),2) & \gamma((1,3),3) \\ \gamma((2,1),1) & \gamma((2,1),2) & \gamma((2,1),3) \\ \gamma((2,2),1) & \gamma((2,2),2) & \gamma((2,2),3) \\ \gamma((2,3),1) & \gamma((2,3),2) & \gamma((2,3),3) \\ \gamma((3,1),1) & \gamma((3,1),2) & \gamma((3,1),3) \\ \gamma((3,2),1) & \gamma((3,2),2) & \gamma((3,2),3) \\ \gamma((3,3),1) & \gamma((3,3),2) & \gamma((3,3),3) \\ \gamma((4,1),1) & \gamma((4,1),2) & \gamma((4,1),3) \\ \gamma((4,2),1) & \gamma((4,2),2) & \gamma((4,2),3) \\ \gamma((4,3),1) & \gamma((4,3),2) & \gamma((4,3),3) \end{bmatrix} = \begin{bmatrix} 3 & 5 & 7 \\ 3 & 4 & 4 \\ 5 & 7 & 9 \\ 7 & 6 & 4 \\ 4 & 5 & 7 \\ 4 & 8 & 6 \\ 6 & 3 & 5 \\ 3 & 7 & 7 \\ 3 & 5 & 4 \\ 4 & 7 & 9 \\ 5 & 6 & 8 \\ 3 & 5 & 4 \end{bmatrix} \tag{24}$$

$$\mathbf{T} = [\tau((i,j),s)] = \begin{bmatrix} \tau((1,1),1) & \tau((1,1),2) & \tau((1,1),3) \\ \tau((1,2),1) & \tau((1,2),2) & \tau((1,2),3) \\ \tau((1,3),1) & \tau((1,3),2) & \tau((1,3),3) \\ \tau((2,1),1) & \tau((2,1),2) & \tau((2,1),3) \\ \tau((2,2),1) & \tau((2,2),2) & \tau((2,2),3) \\ \tau((2,3),1) & \tau((2,3),2) & \tau((2,3),3) \\ \tau((3,1),1) & \tau((3,1),2) & \tau((3,1),3) \\ \tau((3,2),1) & \tau((3,2),2) & \tau((3,2),3) \\ \tau((3,3),1) & \tau((3,3),2) & \tau((3,3),3) \\ \tau((4,1),1) & \tau((4,1),2) & \tau((4,1),3) \\ \tau((4,2),1) & \tau((4,2),2) & \tau((4,2),3) \\ \tau((4,3),1) & \tau((4,3),2) & \tau((4,3),3) \end{bmatrix} = \begin{bmatrix} 0,96 & 1,20 & 1,92 \\ 1,60 & 2,00 & 3,20 \\ 1,50 & 1,88 & 3,00 \\ 1,20 & 1,50 & 2,40 \\ 1,00 & 1,25 & 2,00 \\ 1,30 & 1,63 & 2,60 \\ 0,45 & 0,56 & 0,90 \\ 0,90 & 1,13 & 1,80 \\ 0,70 & 0,88 & 1,40 \\ 1,50 & 1,88 & 3,00 \\ 0,80 & 1,00 & 1,60 \\ 1,60 & 2,00 & 3,20 \end{bmatrix} \tag{25}$$

$$\mathbf{\Delta} = [\delta(i,j)] = \begin{bmatrix} \delta(1,1) & \delta(1,2) & \delta(1,3) \\ \delta(2,1) & \delta(2,2) & \delta(2,3) \\ \delta(3,1) & \delta(3,2) & \delta(3,3) \\ \delta(4,1) & \delta(4,2) & \delta(4,3) \end{bmatrix} = \begin{bmatrix} 96 & 160 & 150 \\ 120 & 100 & 130 \\ 45 & 90 & 70 \\ 150 & 80 & 160 \end{bmatrix} \tag{26}$$

$$\mathbf{\Theta} = \{\theta(1) = 8;\ \theta(2) = 10;\ \theta(3) = 12\} \tag{27}$$

$$\mathbf{A} = \{\alpha(1) = 200;\ \alpha(2) = 100;\ \alpha(3) = 100; \alpha(4) = 200\} \tag{28}$$

$$\mathbf{B} = \{\beta(1) = 150;\ \beta(2) = 230;\ \beta(3) = 220\} \tag{29}$$

The above-mentioned data were introduced into the W.O.Z.T. application. Then calculations were made. As a result of these calculations, efficient solutions were received. The solutions are presented in the table (Fig. 3).

5. VERIFICATION OF THE ALGORITHM FOR METHOD OF SOLVING MULTICRITERIA TRANSPORTATION PROBLEMS IN THE CONTEXT OF MINIMIZING THE NUMBER OF MODES OF TRANSPORT

Verification of the algorithm for method of solving multicriteria transportation problems in the context of minimizing the number of means of transport was made in the W.O.Z.T. application. In this application, a number of tests confirming the correctness of the algorithm have been made. These tests were performed for one hundred times on a mobile computer with the following processor's parameters: Intel® Core™ i5-2520M, 4GB RAM.

EFFICIENT SOLUTIONS:

Solution 1

Values of Criterias:	f1(n)	f2(n)	f3(n)
	61	26432	99,34

	n((1,1)1)	n((1,2)3)	n((2,2)2)	n((3,3)3)	n((4,2)2)	n((4,3)3)
Nr of transport resources	19	5	10	9	8	10
Volume of cargo	150	50	100	100	80	120

Solution 4

Values of Criterias:	f1(n)	f2(n)	f3(n)
	54	28315	136,8

	n((1,2)3)	n((2,1)3)	n((3,1)2)	n((3,3)3)	n((4,2)2)	n((4,3)3)
Nr of transport resources	17	9	5	5	3	15
Volume of cargo	200	100	50	50	30	170

Solution 2

Values of Criterias:	f1(n)	f2(n)	f3(n)
	56	27416	122,5

	n((1,1)1)	n((1,2)3)	n((2,1)3)	n((3,3)3)	n((4,2)2)	n((4,3)3)
Nr of transport resources	7	13	9	9	8	10
Volume of cargo	50	150	100	100	80	120

Solution 5

Values of Criterias:	f1(n)	f2(n)	f3(n)
	68	28032	83,34

	n((1,1)1)	n((1,2)1)	n((2,3)1)	n((3,3)3)	n((4,2)2)	n((4,3)3)
Nr of transport resources	19	7	13	9	18	2
Volume of cargo	150	50	100	100	180	20

Solution 3

Values of Criterias:	f1(n)	f2(n)	f3(n)
	66	27872	88,14

	n((1,1)1)	n((1,2)3)	n((2,3)1)	n((3,3)3)	n((4,2)2)	n((4,3)3)
Nr of transport resources	19	5	13	9	18	2
Volume of cargo	150	50	100	100	180	20

Solution 6

Values of Criterias:	f1(n)	f2(n)	f3(n)
	64	26632	99,84

	n((1,1)1)	n((1,2)3)	n((2,2)1)	n((3,3)3)	n((4,2)2)	n((4,3)3)
Nr of transport resources	19	5	13	9	8	10
Volume of cargo	150	50	100	100	80	120

Solution 7

Values of Criterias:	f1(n)	f2(n)	f3(n)
	62	28132	94,64

	n((1,1)1)	n((1,2)3)	n((2,3)3)	n((3,3)3)	n((4,2)2)	n((4,3)3)
Nr of transport resources	19	5	9	9	18	2
Volume of cargo	150	50	100	100	180	20

Solution 8

Values of Criterias:	f1(n)	f2(n)	f3(n)
	67	28032	86,54

	n((1,1)1)	n((1,2)3)	n((2,3)1)	n((3,3)3)	n((4,2)2)	n((4,3)1)
Nr of transport resources	19	5	13	9	18	3
Volume of cargo	150	50	100	100	180	20

Solution 9

Values of Criterias:	f1(n)	f2(n)	f3(n)
	69	28192	81,74

	n((1,1)1)	n((1,2)1)	n((2,3)1)	n((3,3)3)	n((4,2)2)	n((4,3)1)
Nr of transport resources	19	7	13	9	18	3
Volume of cargo	150	50	100	100	180	20

Fig. 3. Table of efficient solutions generated in W.O.Z.T. application
Source: own study.

For the above numerical example, the W.O.Z.T. application generated nine efficient solutions. The obtained solutions differ from each other by the values of criteria and the number of means of transport directed to particular routes. The generated solutions also contain information about the volume of cargo. For example, in solution No. 4, 17 means of transport (third type) were directed on the route from the first source to the second destination. These modes of transport will conduct a transport of 200 cargo load units. Furthermore, in solution No. 4, the criteria minimizing the number of means of transport adopts the smallest value of all obtained efficient solutions – 54 [pcs.]. However, in this solution, the value of the cost criterion (28 315 [PLN]) and duration criterion (136.8 [h]), have the highest values of all generated efficient solutions.

Tests, aimed in verifying the correct operation of the W.O.Z.T. application, were divided into groups that were differentiated in terms of the number of variables and the number of criteria. Taking into account 100 tests, the final solutions in the form of a set of efficient solutions were generated for 83 tasks. The process of solving 17 remaining tasks was interrupted due to the very long waiting time for the solution (over 1 hour). The longtime of solving these tasks could be connected with the parameters of the computer, on which the tests were performed – the search for solutions was abandoned due to the ineffective time of obtaining them (expectations of a potential group of software customers, i.e. carriers, who have a short waiting time for transport plans, were taken into account).

The obtained results enabled to notice that the task solving time increases with the increase of the number of criteria and the number of variables. On the other hand, the number of received efficient solutions depends on the number of criteria. The greater the number of criteria was, the larger the set of efficient solutions was as well (Fig. 4). The number of variables does not affect the number of efficient solutions.

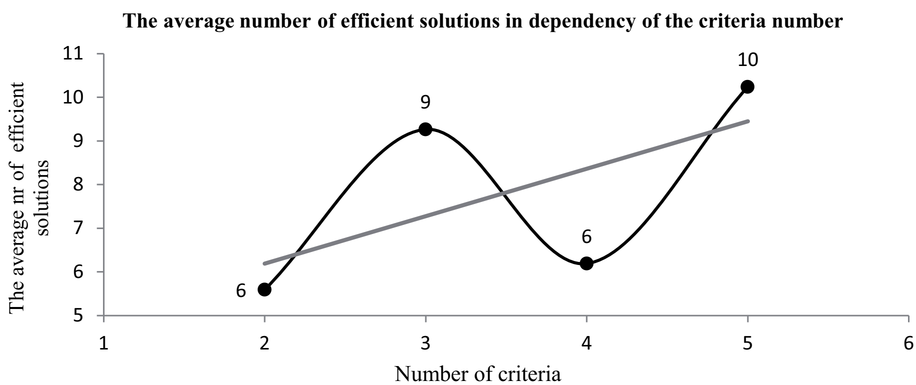

Fig. 4. Graph of the average number of efficient solutions generated in the W.O.Z.T. application (for 83 tests) in dependency of number of criteria
Source: own study

SUMMARY

Multicriteria transportation problems in the context of minimizing the number of means of transport are an alternative approach to transportation problems. Thanks to the introduction of the number of means of transport and their types to formulation of the problem, the transportation problem is considered in a comprehensive manner. The method of solving multicriteria transportation problems in the context of minimizing modes of transport (presented in the article) enables to generate efficient solutions constituting (Pareto)

optimal transportation plans in division into the types of means of transport. This allows to avoid the use of multicriteria optimization methods twice, like it happens for the problems in which the volume of cargo is determined first, and then the appropriate modes of transport are assigned. Besides, thanks to the W.O.Z.T. application, generated solutions in the form of transportation plans, in a simple way and in a short time can be implemented in real cases and applied in many transportation companies. Efficient sets generated in the application can be sets of alternative solutions. Then the choice of the final solutions can constitute a multicriteria problem, requiring the use of additional optimization tools. For this reason, further work should be carried out in order to develop and modify the W.O.Z.T. application in the direction of applying an additional multicriteria assessment method, facilitating the selection of a final solution.

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ANALIZA WIELOKRYTERIALNYCH ZADAŃ TRANSPORTOWYCH W ASPEKCIE MINIMALIZACJI LICZBY STOSOWANYCH ŚRODKÓW TRANSPORTU NA WYBRANYM PRZYKŁADZIE

Streszczenie: W artykule przedstawiono metodę formułowania wielokryterialnych zadań transportowych w kontekście analizy liczb środków transportu. Środki transportu w opracowanej metodzie rozpatrywane są w aspekcie minimalizacji ich liczby. W artykule przedstawiono metody rozwiązywania wielokryterialnych zadań transportowych, a następnie opracowano rozwiązania wielokryterialnych liniowych zadań transportowych w kontekście minimalizacji liczby środków transportu. Opisany został także przykład rozwiązania w efekcie zastosowania autorskiej aplikacji komputerowej. Pewien szczególny nacisk położony został na opracowanie weryfikacji poprawności działania przedstawionego algorytmu. Artykuł został zwieńczony podsumowaniem wraz ze wskazaniem dalszych prac nad zagadnieniem.

Słowa kluczowe: transport, zagadnienie transportowe, badania operacyjne, wielokryterialne wspomaganie decyzji, optymalizacja wielokryterialna